On the effect of substrate properties on the indentation behaviour of coated systems

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Abstract

The indentation behaviour of an elastoplastic coating–substrate system is investigated using a combination of dimensional and finite element analyses. Scaling functions relating the indentation load–depth curves to coating and substrate mechanical properties are given. Based on these scaling functions, the indentation behaviour of various coated systems is examined. The critical indentation depth to coating thickness ratio below which the substrate material has a negligible effect on the indentation response of the coated system is identified for various generic coating–substrate systems. Such ratio is given in terms of the yield strength and Young’s modulus of the coating and substrate, i.e. $\sigma_y/c$ and $E_c/E_s$. The results of parametric studies revealed that the commonly used rule that the maximum indentation depth should be less than 10% of the coating thickness, is applicable only when $\sigma_y/c < 10$. However, indentation experiments should be carried out up to a maximum depth of 5% of the film thickness to avoid any influence from the substrate when $\sigma_y/c \geq 10$ and $E_c/E_s > 0.1$.

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1. Introduction

In controlled indentation tests, an indenter is pressed onto the surface of the material with a prescribed load and either the penetration depth or the radius of the contact area is measured [1]. In ductile materials, a small region underneath the indenter deforms plastically, and the extent of this deformation can be related to both the shape of the indenter and the stress–strain behaviour of the material. Common indenter geometries range from spherical (Brinell), to conical (Rockwell) to four and three-sided pyramids (Vickers or Berkovich). Nano-indenters, which are generally of the Berkovich type, are in particular used to measure the properties of very small volumes of materials [2,3], where loads are of the order of 100 $\mu$N and penetration depths extend to just a few nanometers.

Micro- and nano-indentation techniques have been increasingly used to determine the mechanical properties of substrates and thin films [4–6]. For instance, Oliver and Pharr [5] first proposed the now widely used method to obtain the elastic modulus from the unloading part of the indentation load–depth curves and the hardness from indentation depth measurements. Cheng and Cheng [7,8] relied on finite element techniques to extract the relationships between hardness, contact area, initial unloading slope and mechanical properties of elastoplastic substrates. Tunvisut et al. [9,10] employed dimensional analyses and finite element techniques to determine the relations between the Young’s modulus, yield strength and strain hardening exponent of elastoplastic coatings. A major assumption in that work was that the substrate remained elastic during the indentation process. However, in cases where the coating and substrate materials exhibit comparable strengths, such assumption may not be appropriate as the plastic deformation in the substrate can influence the shape of the indentation curve and, hence, the predicted properties. To extract the mechanical properties of a coating from an indentation curve which has not been affected by the plastic deformation of the substrate, the plastic zone under the indenter should be confined within the coating. If, however, the plastic zone underneath the indenter extends up to the coating–substrate interface, the measured indentation response will no longer be representative of that of the coating alone. In common practice, it is generally assumed that a maximum indenta-
tion depth of one-tenth of the coating thickness should be sufficient to avoid the indentation curve being affected by the plastic deformation of the substrate [9,11]. However, the one-tenth rule has not been fully verified for realistic ranges of coating–substrate properties, in particular in cases where the coating is stronger than the substrate.

In this work, this problem is addressed by investigating the indentation of an elastoplastic coating deposited on an elastoplastic substrate. The scaling functions previously derived by Tunvisut et al. [9] to identify the functional relationship between the indentation curve parameters and the mechanical properties of thin coatings from micro-indentation tests are now extended to account for the additional effects of the substrate’s yield strength and strain hardening exponent, $n_s$. Similarly, the elastoplastic properties of the substrate are described by the Young’s modulus, $E_s$, yield strength, $\sigma_{ys}$, Poisson’s ratio, $\nu_s$, and strain hardening exponent, $n_s$. Both materials are assumed to obey the von Mises yield criterion. It is also considered that there is no friction at the interface between the indenter and the coating.

Unlike the indentation of an elastic- or rigid-plastic solid, the indentation of elastoplastic materials cannot be solved analytically [1]. Instead, the indentation behaviour of an elastoplastic solid using a conical indenter can be studied through a combination of dimensional and finite element analyses. A typical indentation curve (see Fig. 1(b)) is characterised by the loading and unloading parts, the maximum indentation load, $F_{\text{max}}$, its corresponding maximum indentation depth, $h_m$, and the final depth after unloading, $h_f$.

During loading, the indentation load $F$ should be a function of all the independent parameters of the problem, namely, the coating and substrate material parameters ($E_i$, $\sigma_{yi}$, $n_i$, $\nu_i$, $E_s$, $\sigma_{ys}$, $n_s$, $\nu_s$) and the geometric parameters including the coating thickness ($h_c$), the indentation depth ($h$) and the indenter half angle ($\theta$). Thus

$$F = \tilde{f}(E_i, \sigma_{yi}, n_i, \nu_i, E_s, \sigma_{ys}, n_s, \nu_s, h, h_c, \theta).$$

Similarly, the critical indentation depth, $h_c$, below which the influence of substrate is negligible, can be expressed as a function of a similar set of independent parameters:

$$h_c = \tilde{k}(E_i, \sigma_{yi}, n_i, \nu_i, E_s, \sigma_{ys}, n_s, \nu_s, h_m, \theta).$$

Henceforth, $E_s$ and $h_c$ will be assumed to be known for simplicity. Then, by applying the Buckingham II Theorem [12] and choosing the dimensionless quantities so that unknown parameters are normalised by known quantities, the following dimensionless relationships can be obtained:

$$\frac{F}{E_i \sigma_{yi}^2} = \prod_{i} \left(\frac{h}{h_m} \frac{E_i}{E_s} \frac{\sigma_{yi}}{\sigma_{ys}} \frac{n_i}{n_s} \frac{\nu_i}{\nu_s} \theta \right).$$

![Fig. 1](image-url)


\[
\frac{h_i}{h_0} = \hat{\prod}_{\alpha} \left( \frac{E_i}{E_s}, \frac{\sigma_{\alpha}}, \frac{\tau_{\alpha}}, \alpha, \beta, \gamma, \delta \right)
\]

(6)

where \( \hat{\prod}_{\alpha} \) and \( \hat{\prod}_{\beta} \) are dimensionless scaling functions of the indentation load and the critical indentation depth, respectively. It is noted that the inclusion of these dimensionless scaling functions is essential to motivate the choice of parameters to be considered in the subsequent parametric studies.

The main challenge in the approach followed by this work resides in identifying the functional dependence of Eqs. (5) and (6)’s dimensionless functions \( \hat{\prod}_{\alpha} \) and \( \hat{\prod}_{\beta} \) through parametric studies. Next, the numerical procedures used to carry out these studies are discussed and results for the dimensionless functions \( \hat{\prod}_{\alpha} \) and \( \hat{\prod}_{\beta} \) over the range of interest are presented.

3. Finite element procedure

The finite element analyses of the indentation problem were performed using the commercial finite element code ABAQUS [13]. The indentation problem was modelled as a cylinder with a thin coating on its top surface, and with the coating being indented by a rigid conical indenter, as shown in Fig. 2. It can be seen that both the loading and the geometry of the coating-substrate system exhibit a symmetry about the axis of the indenter. The corresponding finite model of the two-dimensional axisymmetric problem is shown in Fig. 3. Here, the angle for the indenter of \( \theta = 70.3^\circ \) was chosen so as to give the same contact area-to-depth ratio as the Berkovich three-sided pyramidal and Vickers four-sided pyramidal indenters [9]. A fine finite element mesh is used in the vicinity of the contact region to capture the localised nature of the local deformation underneath the indenter. A gradually coarser mesh is used away from the contact region in order to reduce the total number of degrees of freedom and computational time. The results of a mesh sensitivity study revealed the 3093 four-noded axisymmetric mesh shown in Fig. 3 to be sufficient to ensure a mesh independent solution for the range of conditions investigated. For the present study, 32 elements were used through the coating thickness. During the analysis, the rigid indenter was loaded by means of a downward displacement along the \( x_2 \) axis (see Fig. 3). The loading and unloading curves were obtained directly from the finite element results given by the normal reaction force on the rigid indenter as a function of the vertical displacement of the node directly beneath the tip of the indenter. Simulations were undertaken for a vari-

![Fig. 2. Geometry of indentation problem (not to scale).](image)

![Fig. 3. Finite element mesh of the axisymmetric indentation problem.](image)
ety of coating and substrate materials characterised by the following property ranges, $0.1 \leq \frac{E_c}{E_s} \leq 5.0$, $0.001 \leq \frac{\sigma_{yc}}{E_s} \leq 0.1$ and $0.001 \leq \frac{\sigma_{ys}}{E_s} \leq 0.1$. For simplicity, the substrate’s Young’s modulus is assumed to be 200 GPa, and both coating’s and substrate’s Poisson’s ratio to be 0.3.

4. Results of the parametric studies

The results of the parametric finite element studies will first be presented for the case where both the coating and the substrate materials exhibit elastic-perfectly plastic behaviour. This will give an insight into how the plastic deformation in the substrate influences the shape of the indentation curve and, hence, the predicted properties. Then, the effect of the substrate strain hardening behaviour on the coating–substrate indentation response is discussed. For brevity, detailed results will only be presented for four typical Young’s modulus ratios, namely, $\frac{E_c}{E_s} = 0.1$, 1.0, 2.0 and 5.0.

4.1. Indentation of an elastic-perfectly plastic coating–substrate system

The functional dependency between the indentation load and the coating and substrate mechanical properties, given by $\hat{\Pi}$ in Eq. (5), is first examined for an elastic-plastic coating–substrate system exhibiting a perfectly plastic behaviour. Fig. 4(a)–(d) show the predicted normalised indentation load, $\frac{F}{h^2 E_s}$, as a function of the normalised indentation depth, $\frac{h}{h_o}$, for different coating yield strengths, $\frac{\sigma_{yc}}{E_s}$, elastic ratios, $\frac{E_c}{E_s}$, and a substrate yield strength of $\frac{\sigma_{ys}}{E_s} = 0.00265$. One can see clearly from Fig. 4(c) and (d) that there are clear inflections in the loading curves at the highest $\frac{\sigma_{yc}}{E_s}$ and $\frac{E_c}{E_s}$ values. This characteristic, which is not observed in the indentation of uncoated materials, indicates that the indentation behaviour of the coating has been affected by the plastic deformation of the substrate.

The information presented in Fig. 4(a)–(d) is also re-plotted in Fig. 5 in terms of the normalised maximum indentation load, $\frac{F_{\text{max}}}{h^2 E_s}$, versus the normalised coating yield strength, $\frac{\sigma_{yc}}{E_s}$, for $\frac{E_c}{E_s} = 0.1$, 1.0, 2.0 and 5.0 and the chosen normalised substrate yield strength. It can be seen that the maximum indentation load increases continuously with increasing $\frac{\sigma_{yc}}{E_s}$ and $\frac{E_c}{E_s}$ ratios. This increase becomes weaker when the coating is more compliant than the substrate.

Fig. 6 shows the relative effect of the plastic deformation of the substrate on the normalised maximum indentation load versus coating yield strength curves. Here, the absence of plastic deformation in the substrate causes the maximum
indentation load to increase significantly with respect to the elastic–perfectly plastic substrate case for $E_c/E_s > 1.0$.

This implies that the predicted properties based on Tunvisut et al.'s [9] method would be overestimated for $E_c/E_s > 1.0$ if the substrate is assumed to behave elastically. The effect of the substrate yield strength on the indentation behaviour is given in Fig. 7(a)–(d). They show the computed normalised indentation load, $F/h^2E_s$, as a function of the normalised indentation depth, $h/b_0$, for different substrate yield strengths, $\sigma_{yc}/E_s$, elastic ratios, $E_c/E_s$, and a coating yield strength of $\sigma_{yc}/E_s = 0.01$. For a given $E_c/E_s$ ratio, it is seen that the loading curves for all $\sigma_{yc}/E_s$ ratios are initially identical but start to deviate at certain depths depending on the elastic mismatch. These results highlight the strong effect that the plastic deformation in the substrate can have on the indentation behaviour of the coated system. In Fig. 8, the normalised maximum indentation loads, $F/h^2E_s$, from Fig. 7(a)–(d) are shown as a function of the normalised substrate yield strength, $\sigma_{yc}/E_s$. These results reveal that, for the coating yield strength of $\sigma_{yc}/E_s = 0.01$, the normalised maximum indentation load is relatively insensitive to the normalised substrate yield strength once $\sigma_{yc}/E_s > 0.01$. Note that, the maximum indentation load is controlled mostly by the initial coating yield strength since the substrate still behaves elastically, i.e. $\sigma_{yc} \gg \sigma_{ys}$. However, the effect of substrate yielding is evident when $\sigma_{yc}/E_s < 0.01$, leading to a relaxation of the maximum indentation load.

Fig. 9(a)–(e) give the contour plots of accumulated equivalent plastic strain at different indentation depths for the coating–substrate system with $E_c/E_s = 5.0$ and $\sigma_{yc}/\sigma_{ys} = 20$. It can be seen that plastic deformation in the substrate initiates when the penetration depth is between 8 and 15% of the coating thickness. The accumulated equivalent plastic strain increases with increasing penetration depths resulting in the inflections and deviations in the loading part of the indentation curve shown previously in Figs. 4(c)–(d) and 7(a)–(d).

4.2 Indentation of a strain hardening elastic–plastic coating–substrate system

In this section, the effect of substrate strain hardening on the indentation response of the coating–substrate system is examined. The effect of coating strain hardening on the indentation behaviour has been studied previously in the work of Tunvisut et al. [9]. Additionally, it was found that, for high values of $\sigma_{yc}/E_s$, the work hardening exponent of the coating has little effect on the indentation curve when the coating is more compliant than the substrate, i.e. $E_c/E_s < 1.0$. In this work, the coating is simply assumed to be elastic–perfectly plastic so that the dependence of substrate strain hardening on the indentation response can be uniquely determined. It is noted that similar results would be expected when the coating work hardening behaviour is taken into account. The dimensionless function $\bar{F}_c$, as given in Eq. (5), is examined for substrate materials exhibiting elastic–perfectly plastic ($\sigma_s = \infty$), moderate ($\sigma_s = 10$) and strong hardening ($\sigma_s = 5$) behaviour. Simulations were carried out for $\sigma_{yc}/E_s = 0.001, 0.01$ and $E_c/E_s = 0.05$ and $E_c/E_s = 0.1$ and $E_c/E_s = 2.0$ with a chosen value of $\sigma_{yc}/E_s = 0.00265$. The results are presented in Fig. 10(a) and (b) for $E_c/E_s = 0.1$ and 2.0, respectively. It can be seen that, for low values of $\sigma_{yc}/E_s$, the substrate work hardening exponent has negligible effect on the indentation behaviour for both $E_c/E_s = 0.1$ and 2.0. However, at high values of $\sigma_{yc}/E_s$ and deep indentations with respect to the coating thickness, this effect becomes more apparent.
Fig. 7. Effect of the normalised substrate yield strength ($\sigma_{ys}/E_s$) on the indentation response of a coating with $\sigma_{yc}/E_s = 0.01$ and an elastic mismatch ($E_c/E_s$) of (a) 0.1, (b) 1.0, (c) 2.0 and (d) 5.0.

4.3. Determination of the critical indentation depth to coating thickness ratio

When an indentation is made on a coating–substrate system, the deformation process will be quite different from that on a bulk material. The overall indentation response is determined by the interaction of the stress and strain fields from the coating and substrate. The influence of the substrate becomes more significant with increasing indentation depth. This fact can make the determination of coating properties and its hardness measurement dubious or even completely wrong. It is therefore advantageous to know for a given coating–substrate system a range of indentation depth which can be made so that the influence of the substrate is minimal. In order to investigate the influence of the substrate, the dimensionless function $\hat{\beta}$, given in Eq. (6), is studied next.

Finite element results are presented in Fig. 11 showing the critical indentation depth to the coating thickness ratio ($h_c/h_o$), beyond which the substrate material deforms plastically, as a function of the coating–substrate yield strength ratio for the elastic-perfectly plastic coating–substrate systems with $E_c/E_s = 0.1, 0.5, 1.0$ and 5.0. It can be seen that the critical indentation ratio decreases with increasing yield strength ratio and elastic mismatch. However, the dependence of the critical ratio on the yield strength and Young’s modulus ratios becomes insignificant when $E_c/E_s > 0.5$ and $\sigma_{yc}/\sigma_{ys} > 10$. In this figure, it can also be inferred that the commonly used one-tenth rule is adequate in these cases for all ranges of $E_c/E_s$, provided that $\sigma_{yc}/\sigma_{ys} < 10$. However, this rule is not appropriate for coated systems with

Fig. 8. Relationship between the normalised maximum indentation load and the normalised substrate yield strength for an elastic-perfectly plastic coating–substrate system.
higher $\sigma_{yc}/\sigma_{ys}$ values. For instance, a maximum indentation depth of about 5% of the coating thickness would only be feasible when $\sigma_{yc}/\sigma_{ys} > 20$ and $E_{c}/E_{s} > 0.1$. Thus the commonly used one-tenth rule would be invalid in situations when a very hard coating (e.g. Mo with a typical yield strength greater than 2 GPa) is deposited on a soft substrate (e.g. mild steel with a typical yield strength in the order of 200 MPa). In such situations, the relatively harder coating allows the plastic deformation to develop easily in the substrate when the hard coating is indented. In such cases, the substrate material experiences greater plastic deformation than in cases of much smaller yield strength mismatches. These findings are also consistent with the recent work of He and Veprek.

The effect of the coating strain hardening behaviour on the critical indentation depth to coating thickness ratio, $h_{c}/h_{o}$, is next examined. It is worth pointing out that the effect of substrate strain hardening behaviour on the critical indentation depth to coating thickness ratio is negligible when the substrate does not yield. Fig. 12 shows the effect of the coating strain hardening exponent $n_{c}$ on $h_{c}/h_{o}$, which in fact is negligibly small. This observation suggests that the function
\[ \prod_{\beta} \text{in Eq. (6)} \] can be further simplified, by removing \( n_c \) and \( n_s \) as independent parameters. Furthermore, assuming that \( \nu_c, \nu_s \) and \( \theta \) are fixed, then Eq. (6) simplifies to
\[ h_c / h_o = \prod_{\beta} \left( \frac{E_c}{E_s}, \frac{\sigma_{yc}}{\sigma_{ys}} \right). \]

In order to enhance the applicability of the findings in Figs. 11 and 12, a functional relation for \( \prod_{\beta} \) was fitted to the numerical results. A suitable expression for the critical indentation depth to coating thickness ratio was found to be;
\[ h_c / h_o = \exp^{-1.695+0.501/\bar{\sigma} -0.444/\bar{E} -0.065/\bar{E} - 0.170 \ln \bar{E}}, \]

where \( \bar{\sigma} = \sigma_{yc}/\sigma_{ys} \) and \( \bar{E} = E_c/E_s \). The trends predicted by Eq. (8) are shown in Fig. 11.

4.4. Influence of the substrate on hardness

Hardness is generally defined as the mean pressure supported by the material under the indentation load [6].
\[ H = \frac{F_{\text{max}}}{A}, \]

where \( F_{\text{max}} \) is the peak indentation load and \( A \) is the projected contact area at peak load. Here, the value of \( A \) as well as \( F_{\text{max}} \), will be determined from the FE results.

Fig. 13 shows the computed hardness for the elastic-perfectly plastic coating–substrate system as a function of coating–substrate yield strength ratios with three values of maximum allowable substrate accumulated equivalent plastic strain, i.e. \( \tilde{\varepsilon}_{ps} \), with \( E_c/E_s = 1.0 \) and \( \sigma_{yc} = 2000 \text{MPa} \). The results shown in this figure were obtained for a constant coating yield strength of \( \sigma_{yc} = 2000 \text{MPa} \). It is apparent from this figure that the plastic deformation in the substrate only significantly affects the predicted hardness at relatively deep indentations due to large plastic deformation in the substrate. This effect is relatively weak when \( \sigma_{yc}/\sigma_{ys} < 10 \) but becomes more significant when \( \sigma_{yc}/\sigma_{ys} > 10 \). This figure also reveals that the predicted coating hardness with the maximum of 2% equivalent total plastic strain in the substrate, \( \tilde{\varepsilon}_{ps} = 0.02 \), deviates by less than 5% from that predicted with an elastic substrate, \( \tilde{\varepsilon}_{ps} = 0 \). Thus, a slight plastic deformation in the substrate may not lead to significant differences in the predicted hardness since in most hardness measurements the scatter is approximately 5%. Therefore, it can be said that the influence of the substrate plastic deformation on the predicted hardness can be assumed to be negligible when the maximum equivalent total plastic strain anywhere in the substrate
is less than about 2%. These results mean that the maximum indentation depth, when \( \sigma_{yc}/\sigma_{ys} > 20 \) and \( E_c/E_s > 0.1 \), can be greater than 5% of the coating thickness without being significantly affected by the substrate if one is only interested in obtaining the hardness of the coating. Furthermore, from systematic studies conducted as part of this work, it was found that, when measuring the hardness of a coating–substrate system, the stronger and less compliant the coating is, the more severe the influence of the substrate will be. It should also be noted that the substrate influence usually leads to lower values of the measured hardness.

4.5. Effect of the substrate plastic deformation on the critical indentation depth to coating thickness ratio

Fig. 14(a)–(d) show the critical ratios of indentation depth to coating thickness, \( h_c/h_o \), as a function of yield strength ratio, \( \sigma_{yc}/\sigma_{ys} \), for different elastic ratios and various degrees of plastic deformation in the substrate, \( \varepsilon_p^{ps} \). For the range of interest of \( E_c/E_s \), these results show that the decrease of \( h_c/h_o \) with \( \sigma_{yc}/\sigma_{ys} \) is sharp at the beginning, when the yield strength ratio is relatively low, i.e. \( \sigma_{yc}/\sigma_{ys} < 5 \), and then becomes approximately constant for greater yield strength ratios, i.e. \( \sigma_{yc}/\sigma_{ys} > 20 \). Furthermore, the critical indentation ratio is strongly dependent on \( \varepsilon_p^{ps} \). For example, when a maximum of 2% of accumulated plastic strain in the substrate is allowed, the critical ratio increases considerably, that is a deeper indentation as a fraction of coating thickness could be made. The trends shown in Fig. 14(a)–(d) provide approximate values of the \( h_c/h_o \) ratios for a range of yield strength and elastic ratios. Hence, they can also be used as a guideline for approximating how deep indentation experiments can be made in order to avoid the influence of the substrate in the coated systems with known mechanical properties. For example, when \( E_c/E_s = 0.5 \), the value of \( h_c/h_o \) to avoid any substrate effects decreases from 0.26 for \( \sigma_{yc}/\sigma_{ys} = 1.67 \) to 0.07 for \( \sigma_{yc}/\sigma_{ys} = 20 \), see Fig. 14(b). Clearly, the commonly used one-tenth rule overestimates the critical indentation ratio and, hence, would not be applicable in this case. However, in any indentation experiments, it should be worth noting that the indentation depth should be at least 0.3 \( \mu m \) so that the possibility of indentation size effect is avoided [15–17].

5. Conclusions

The effect of substrate properties on the indentation behaviour of coated systems has been investigated using a combination of dimensional and finite element analyses.
Parametric studies show that there are clear inflections and deviations within the loading part of the indentation curves caused by the plastic deformation in the substrate. Such substrate effects can be large and must therefore be taken into consideration during indentation analyses. From the present results, it also follows that the commonly used one-tenth rule to estimate the indentation depth beyond which substrate effects cannot be ignored is not universally applicable. For a given indenter geometry, it was found that an increase in the coating Young’s modulus and yield strength with respect to the substrate decreases the critical indentation depth to the coating thickness ratio. In the case of a soft coating on a hard substrate, i.e. \( \sigma_{yc}/\sigma_{ys} < 1 \), the relative indentation depth can even exceed 30% of the film thickness without the indentation curve being significantly influenced by the presence of the substrate since plastic deformation is generally confined within the coating. In contrast, when indenting a hard coating applied on a soft substrate, i.e. \( \sigma_{yc}/\sigma_{ys} > 10 \), the maximum indentation depth can be as small as 5% of the coating thickness. Overall, the maximum indentation depth should not exceed 5% of the coating thickness in order to avoid substrate effects on the indentation behaviour of any coating. When this rule is satisfied, one may simply use the method proposed by Tunvisut et al. [10] to evaluate the properties of the coating, viz Young’s modulus, yield strength and strain hardening exponent. Otherwise, an alternative strategy based on FE and inverse analyses is required.

**Acknowledgements**

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**Appendix A. Nomenclature**

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<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>projected contact area at maximum indentation load</td>
</tr>
<tr>
<td>( E_c, E_s )</td>
<td>coating and substrate Young’s moduli, respectively</td>
</tr>
<tr>
<td>( \bar{E} )</td>
<td>Young’s modulus ratio (( \equiv E_c/E_s ))</td>
</tr>
<tr>
<td>( f_l )</td>
<td>functional dependency of indentation load</td>
</tr>
<tr>
<td>( F )</td>
<td>indentation load</td>
</tr>
<tr>
<td>( F_{max} )</td>
<td>maximum indentation load</td>
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<tr>
<td>( h )</td>
<td>indentation depth</td>
</tr>
<tr>
<td>( h_c )</td>
<td>critical indentation depth</td>
</tr>
<tr>
<td>( h_o )</td>
<td>coating thickness</td>
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<tr>
<td>( H )</td>
<td>material hardness</td>
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<tr>
<td>( \hat{k} )</td>
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<tr>
<td>( n_i )</td>
<td>material strain hardening exponent</td>
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<tr>
<td>( n_c, n_s )</td>
<td>coating and substrate strain hardening exponents, respectively</td>
</tr>
<tr>
<td>( \bar{E} )</td>
<td>Young’s modulus ratio (( \equiv E_c/E_s ))</td>
</tr>
<tr>
<td>( \bar{\sigma} )</td>
<td>yield strength</td>
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<tr>
<td>( \bar{\sigma}<em>{yc}, \bar{\sigma}</em>{ys} )</td>
<td>coating and substrate yield strength, respectively</td>
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<tr>
<td>( \theta )</td>
<td>indenter half angle</td>
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**Greek letters**

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>( \nu_c, \nu_s )</td>
<td>coating and substrate material’s Poisson’s ratio, respectively</td>
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<td>( \epsilon_i )</td>
<td>equivalent strain</td>
</tr>
<tr>
<td>( \epsilon_{yi} )</td>
<td>yield strain</td>
</tr>
<tr>
<td>( \sigma^p )</td>
<td>accumulated equivalent plastic strain</td>
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<tr>
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<td>equivalent stress</td>
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<td>( \hat{\sigma}<em>{yc}, \hat{\sigma}</em>{ys} )</td>
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<tr>
<td>( \bar{\sigma} )</td>
<td>yield strength ratio (( \equiv \sigma_{yc}/\sigma_{ys} ))</td>
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**References**