Thermally induced failure of multilayer ceramic structures

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ABSTRACT

In this work, a methodology is proposed to predict the fracture behaviour of multilayer ceramic systems subjected to thermal shock conditions. The statistical variability exhibited by the fracture behaviour of three-layer structures used in solid oxide fuel cells under transient thermal stresses is determined. The dominant fracture mode was found to be unstable crack propagation from existing interfacial defects oriented normal to the plane of the layers. Finite-element analyses and a weight function method are relied upon to obtain analytical solutions for the interfacial crack driving force in two- and three-layer ceramic structures subjected to inhomogeneous in-plane temperature distributions. Failure diagrams are constructed in terms of geometric and loading variables which allow fail-safe regions to be identified. The effect of the elastic property mismatch between the ceramic layers on the maximum interfacial crack driving force is also discussed.

§ 1. INTRODUCTION

The fracture behaviour of thin multilayer ceramic systems has been the focus of intensive research owing to their wide applications in areas such as high-temperature protective coatings (Busso 1999), electronic packaging (Tummala et al. 1992) and ferroelectric actuators (Gong et al. 1996). Despite their excellent high-temperature properties, relatively low thermal conductivity and high elastic stiffness, ceramic materials are highly susceptible to catastrophic failure when subjected to severe temperature gradients owing to their inherent brittleness (Barsoum 1997).

In order to assure the mechanical integrity of multilayer ceramic structures exposed to thermal transients typical of service, predictive tools must be included in a fail-safe design methodology. Generally, fracture of brittle multilayer structures can occur due to the propagation of surface, interfacial or edge defects (Hutchinson and Suo 1992, Evans and Hutchinson 1995). The propagation of pre-existing flaws normal to either the surface or the multilayer interfaces under in-plane thermal gradients is typically controlled by the resulting thermal stresses (for example Schneider and Petzow (1993)). Furthermore, edge or embedded defects lying on the plane of an interface can propagate under the high interfacial tractions induced by property mismatch between the layers, leading to delamination-type failures.

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These different types of defect may coexist together, in which case the failure of the multilayer structure is controlled by the dominant fracture mechanism.

This work deals with the fracture behaviour of two- and three-layer thin ceramic systems under thermal transient conditions. An example of such systems are three-layer structures used as the central component in solid oxide fuel cells (SOFCs) being developed for small-scale power generation (for example Gong et al. (1996)). A generic three-layer system is illustrated in figure 1. These ceramic structures are subjected to large thermal stresses during service as a result of the temperature transients experienced during start-up and shut-down operations. The main objective of this work is to develop a methodology to measure and predict the fracture behaviour of typical two- and three-layer ceramic systems when subjected to inhomogeneous in-plane temperature distributions.

The structure of the paper is as follows. In §2 the details of the experimental thermal shock procedure are presented, and the experimental results summarized in §3. In §4, a fracture mechanics methodology is presented to describe the failure of the multilayer systems from interfacial defects. Finally, discussions and conclusions are presented in §§5 and 6 respectively.

§2. EXPERIMENTAL PROCEDURE

2.1. Specimens and materials

The multilayer test specimens used in the present work are based on 25 mm x 25 mm square samples of fully dense 8% yttria-stabilized zirconia (YSZ).
The two- and three-layer samples were constructed by applying 50 μm thick layers on either one or both sides of the zirconia 200 μm thick plates respectively. The compositions of the outer layers were typical of materials used in SOFC structures, namely a porous Ni/YSZ cermet (to be referred as the anode) and a porous Sr-doped LaMnO₃ layer (to be referred as the cathode). The outer layers were deposited by screen printing and then sintered at 1300°C for 1 h.

Optical observation of the untested YSZ specimens revealed surface defects introduced during polishing of up to 10 mm in surface length, and an average 8 μm grain size. The average grain sizes in the anode and cathode materials were found to be 1.1 and 1.9 μm respectively. In both cases, the average porosity was approximately 10 vol.%. A study of the effect of sintering conditions on grain size and porosity for the anode and cathode materials has been reported by Chandra et al. (2000). A JEOL 5300 scanning electron microscope with image capturing and digital image analysis capabilities was used for the microstructural observations.

The elastic modulus of YSZ was taken as 200 GPa, and the post-sintering moduli for the anode and cathode were determined by Chandra et al. (2000) to be approximately 60 GPa. A value for Poisson’s ratio of 0.3 was assumed for all materials. Values for the thermal expansion coefficients of the YSZ, anode and cathode have been found to be 10.8 × 10⁻⁶, 12.0 × 10⁻⁶ and 12.8 × 10⁻⁶ respectively (Tummala et al. 1992).

### 2.2. Experimental set-up

The fracture behaviour of ceramic materials exhibits considerable statistical variability and depends on the stress state and the volume of the stressed material (Barsoum 1997, Schneider and Petzow 1993). In this work, the statistical effects on the failure of the thin ceramic multilayers under transient thermal stresses are first investigated experimentally. A thermal shock experiment was designed to measure the critical thermal conditions for fracture initiation in the ceramic multilayers under an equal biaxial stress state. This was achieved by allowing each sample to reach a uniform temperature and then forcing a compressed air jet to impinge at its centre. The experimental set-up, consisting of a radiant furnace connected to an external air supply and data acquisition system, is illustrated in figure 2. The temperature in the furnace was allowed first to stabilize at 950°C before the cool air was introduced. The air was supplied through an open-ended ceramic tube inserted vertically into the furnace through its ceiling. The top of the ceramic tube was connected to a pressure-gauge-controlled air supply, and the opposite end was positioned at the specimen centre. Four fine vertical slots were cut into the lower end of the ceramic tube to enable the air to escape radially (see details in the inset of figure 2). The connection of the ceramic tube with the air supply was designed so as to allow the tube to drop vertically at the moment of the specimen fracture, and this was detected from the sudden temperature drop and changes in the linear variable-differential transducer signal. During the experiment, the temperature distribution on the specimen surface was measured by five thermocouples contacting the underside of the specimen. Preliminary measurements of temperature variations revealed that, owing to the symmetry of the air flow, this number of thermocouples was sufficient to approximate the actual temperature distribution accurately. This issue will be discussed in more detail in the next section. Furthermore, temperature variations through the specimen thickness during the thermal shock test were found to be
negligibly small, as expected in view of the very thin nature of the specimens (i.e. 200–300 μm thick).

One-, two- and three-layer samples were subjected to a sudden temperature transient at their centres, with the air flow gradually increasing until fracture occurred. The temperature distribution at fracture was recorded in each case. Fractographic investigations were performed on each specimen to identify dominant fracture patterns and crack initiation sites.

20 YSZ specimens and, owing to limited material availability, six two-layer and three-layer samples were tested under thermal shock. Thus only for the single-layer YSZ case can the number of samples be considered to be statistically meaningful. As will be shown later, this should not restrict the significance of the results due to the dominant role played by the YSZ substrate in the failure of the two- and three-layer samples. Typical fracture patterns of the one-layer and three-layer specimens after

Figure 2. General layout of the thermal shock experimental set-up: LVDT, linear variable-differential transducer.

Figure 3. Typical fracture patterns after thermal shock testing.
3. Results

3.1. Fracture stresses

The thermal stresses which develop during the specimen centre cooling were calculated numerically using finite-element techniques. From the thermocouple measurements, it was established that the temperature distribution on the surface of the specimen at the moment of fracture could be described accurately by the following function:

\[ T = T_0 - \Delta T \left[ 1 - \left( \frac{T}{T_0} \right)^n \right], \]

where \( \Delta T \) is the temperature differential between the ambient temperature \( T_0 = 950^\circ C \) in the furnace and that at the specimen centre, \( L \) is half the specimen width, \( r \) is the radial distance from the specimen centre and \( n \) is an exponent determined from the measured temperature distribution. Figure 4\( (a) \) shows typical temperature transients measured by each of the five thermocouples on one of the single-layer specimens, with time normalized by the time \( t_f \) to fracture. The inset in figure 4\( (a) \) indicates the relative position of each thermocouple on the specimen. Here, the vertical and horizontal broken lines are aligned with the centre of the slots shown in the inset of figure 2. The axisymmetric nature of the temperature distribution at the moment of fracture can be inferred from the temperature differences between thermocouples located at the same distance from the specimen centre but offset by 45° from each other (i.e. the pair 2 and 3 and the pair 4 and 5 in figure 4\( (a) \)). At \( t/t_f = 1 \), the maximum temperature difference found between either thermocouples 2 and 3, or thermocouples 4 and 5 in any of the specimens tested was 4°C (0.4%). Figure 4\( (b) \) shows a comparison between the temperature measurements obtained from thermocouples 1, 2 and 4 at different relative times with the corresponding spatial distributions given by equation (1). Here, \( T_0 = 950^\circ C \), and \( n = 0.5 \). The maximum discrepancy between the data and the fitting of equation (1) for any of the tested specimens was found to be 3.5°C (or 0.35%). It should be pointed out that, originally, a periodic function was chosen to describe the measured temperature profiles, which allowed for temperatures to vary as a function of angle for points located at the same radial distance from the specimen centre. However, this gave increased complexity with no significant advantage over equation (1). As previously discussed, as a result of the thin nature of the samples, temperature variations through the specimen thickness are assumed to be negligible.

A typical contour plot of the temperature distribution on the surface of the one-layer specimen at the moment of fracture is given in figure 5\( (a) \). Here, only a quarter of the sample is shown owing to symmetry. The location of the thermocouples are indicated in figure 5\( (a) \) by the open circles. The corresponding distribution of the maximum principal stresses obtained from a thermoelastic three-dimensional finite-element analysis of the YSZ specimen is shown in figure 5\( (b) \). These results reveal that the region of maximum (tensile) principal stress is located at the centre of the specimen, in agreement with the observed crack initiation sites in all tested samples.
The statistical treatment of the thermal shock results for the YSZ was based on a two-parameter Weibull distribution. Here, the probability \( P_f \) that a volume \( V \) of material would fail under a maximum principal stress \( \sigma \) is given by

\[
P_f = 1 - \exp \left[ - \int_V \left( \frac{\sigma}{\sigma_w} \right)^m \, dV \right],
\]

where \( \sigma_w \) and \( m \) are the Weibull stress and modulus respectively. For YSZ, it was found that \( \sigma_w = 74.5 \) MPa and \( m = 10.8 \).

The average fracture stresses at the centre of both the YSZ samples and the two-layer samples (anode–YSZ and cathode–YSZ) were found to be 73 ± 28 MPa. In Figure 4. Typical measured temperature distributions (a) temperature transients recorded with the different thermocouples, with time normalized by its value at failure \( (t_f) \); (b) comparison between the measured spatial temperature distributions at different relative times with those given by equation (1) with \( n = 0.5 \).
contrast, the average value of the fracture stress at the centre of the three-layer specimens was significantly greater, namely 112 ± 13 MPa.

3.2. Dominant fracture modes

Detailed scanning electron microscopy (SEM) analyses of the fracture surfaces of tested YSZ specimens revealed both transgranular and intergranular types of crack propagation. In addition, it was also noted that transgranular crack growth prevailed in regions close to the specimen centre, where the higher thermal stresses and lower temperatures were found, whereas the intergranular mode appeared dominant far from the fracture initiation sites in the high-temperature regions. Typical fracture surfaces for both types of crack propagation mode in the YSZ are shown in figures 6(a) and (b).

All specimens subjected to thermal shock were found to have failed because of cracking normal to the plane of the specimens (see for example figure 3). SEM observations of the fracture surfaces of the two-layer and three-layer samples also revealed a small number of secondary interfacial cracks along the anode–YSZ interfaces. This can be seen in the SEM picture in figure 7(a), which shows the fracture surface of one of the anode–YSZ samples and a secondary interfacial crack normal to the plane of the picture. It should be noted that the propensity of the anode–YSZ interface to develop delamination-type interfacial cracks such as that shown in figure 7(a) can be related to the known weak sinterability between the anode material and YSZ (Chandra et al. 2000). A typical fracture surface from the cathode–YSZ specimens is shown in figure 7(b), together with the apparent initiation site. No secondary interfacial cracks were found in such samples. SEM observations of the tested three-layer specimens revealed qualitatively the same features as those observed in the two-layer samples.

These results therefore clearly show that failure in all the samples was caused by unstable crack propagation from either surface or interfacial defects introduced into the YSZ during processing. It should be pointed out that the presence of polishing-related defects such as those observed here is generally accepted as part of a typical industrial surface finishing. It is therefore important to ascertain the implications of such a defect-tolerant approach in terms of the mechanical reliability of YSZ-based multilayer structures.
The prediction of the critical thermal shock conditions which can lead to the failure of the multilayer system requires an accurate description of the local driving force to cause the unstable propagation of interfacial defects lying normal to the interface. This aspect is addressed in the next section.

§4. ANALYTICAL TREATMENT OF SUBCLAD INTERFACIAL CRACKS

Consider an interfacial crack, between the top and middle layers of a three-layer system, oriented normal to the interface (hereafter to be referred as a ‘subclad interfacial crack’), as illustrated in figure 8(a). Here, a generic three-layer system is considered to illustrate the approach. However, a generalization to a system with a greater number of layers, or its application to the two-layer system as a special case, can readily be made.

The maximum stress intensity factor (SIF) of a subclad interfacial crack under a known inhomogeneous in-plane temperature distribution depends on the thermoelastic properties of the individual layers \( (E_i, \nu_i \text{ and } \alpha_i, \text{ for } i = 1, 2, 3) \), and the
geometric \((a, c, L \text{ and } h_i, \text{ for } i = 1, 2, 3)\) and temperature distribution \((T_0, \Delta T \text{ and } n)\) parameters:

\[
K_i = \hat{F}_K \{E_i, \nu_i, \alpha_i, h_i, a, c, L, T_0, \Delta T, n\}, \quad \text{for } i = 1, 2, 3. \tag{3}
\]

A parametric solution to equation (3) can in principle be obtained numerically. However, finite-element analysis of two- or three-dimensional cracks is time consuming and costly, particularly when parametric studies are required. There is therefore a strong need for alternative analytical or semianalytical methods. A recent weight-function-based approach was proposed to determine the SIF of subclad cracks in nuclear pressure vessels (Hodulak and Siegele 1995). This approach will
here be extended to address subclad interfacial cracks in multilayer planar brittle structures.

For simplicity, the long interfacial crack of the type shown in figure 8 (a) will first be approximated by an equivalent through-crack of depth equal to that at the deepest crack front location in the semielliptical crack. A correction will then be applied to the through-crack SIF to account for the actual semielliptical nature of the crack. The existence of geometry-dependent weight functions for cracked planar homogeneous systems allows the effects of geometry and load on the SIF to be accounted for separately. When the weight function is known, the calculation of the SIF is reduced to a simple integral and only requires knowledge of the stresses along the crack line in the uncracked body (for example Labbens et al. (1976)). A general expression for the SIF of a cracked homogeneous plate is

\[ K_{10} = \left( \frac{w}{\pi/2} \right)^{1/2} \int_0^{h_i} \frac{m(\bar{z}, \bar{\alpha}) \sigma(\bar{z})}{(\bar{\alpha} - \bar{z})^{1/2}} \, d\bar{z}, \]  

where \( w \) is the plate thickness, \( z \) is the distance from the cracked surface \( \bar{z} = z/w \), \( \bar{\alpha} = a/w \) is the normalized crack depth, \( m(\bar{z}, \bar{\alpha}) \) is the weight function and \( \sigma(\bar{z}) \) is the stress distribution along the crack line in the uncracked system. Weight functions for planar structures have been given by Labbens et al. (1976) (see also Hodulak and Siegele (1995)). An expression for \( m(\bar{z}, \bar{\alpha}) \) is given in appendix A.

When considering an extension of the weight function method to the cracked three-layer system shown in figure 8 (b), an equivalent system with a virtual surface crack of depth \( t = h_1 + a \) needs to be considered (see figure 8 (c)). In this case, equation (4) becomes

\[ K_{10} = \left( \frac{w}{\pi/2} \right)^{1/2} \left( \int_{h_1}^{h_t} \frac{m(\bar{z}, \bar{t}) \sigma_c}{(\bar{t} - \bar{z})^{1/2}} \, d\bar{z} + \int_{h_1}^{h_1} \frac{m(\bar{z}, \bar{t}) \sigma(\bar{z})}{(\bar{t} - \bar{z})^{1/2}} \, d\bar{z} \right), \]  

where \( w = \sum_{i=1}^{3} h_i \), is the overall thickness of the multilayer system (see figure 8 (a)), \( \bar{t} = (h_1 + a)/w \) is now the normalized distance between the outer surface of layer 1 and the crack tip, \( \bar{h}_t = h_t/w \) and \( \sigma(\bar{z}) \) is the stress distribution along the \( X_3 \) axis in the uncracked three-layer system. Furthermore, \( \sigma_c \) is a closure stress acting along the virtual crack line in layer 1 to account for the fact that layer 1 is uncracked (see figure 8 (c)).

An estimate for \( \sigma_c \) can be found by assuming that one half of the force obtained by integrating the tractions removed by the presence of the crack of depth \( a \) is carried by the uncracked ligament in layer 2, and the other half by layer 1. It can be shown (Hodulak and Siegele 1995) that

\[ \sigma_c = -\sigma_m \frac{a}{2h_1} F. \]  

Here, the mean traction \( \sigma_m \) is rather arbitrarily set to be equal to \( \sigma(\bar{z}) \) at \( \bar{z} = (h_1 + a/2)/w \), and \( F \) is a coefficient which is chosen so that the value of the SIF given analytically by equation (5) agrees with that calculated numerically from a three-dimensional finite-element analysis of the cracked three-layer structure.

In general, the SIF for a through-crack of depth \( a \) can be related to that of a long semielliptical crack of length \( c \) and maximum depth \( a \) through a correction factor \( Q \).
\[ Q = 1 + f_1 \left( \frac{a}{c} \right)^{f_2}, \]

where \( f_1 = 1.46 \) and \( f_2 = 1.65 \) (Anderson 1995). Thus the SIF at the deepest point of the semielliptical subclad interfacial crack of figure 8(a) is then given by

\[ K_1 = \frac{K_{10}}{Q}, \]

with \( K_{10} \) as defined in equation (5). The solution to the integral in equation (5) was numerically obtained using Simpson’s rule.

Thermoelastic stress analyses for the cracked and uncracked multilayer systems under the experimentally measured temperature distributions were conducted using a commercial finite-element code (ABAQUS 1995). The resulting stresses from the uncracked system and the SIFs from the cracked finite-element system were then used to calibrate the values of \( F \). In the calculations to be shown next, a typical surface crack length \( c \) of 10 mm was taken in all cases.

§ 5. Discussion

In order to generalize the proposed approach to generic two- and three-layer systems, the dependence of the parameter \( F \) on the elastic property mismatch between the different layers was investigated. The calibrated \( F \) values are given in figures 9(a) and (b) for the two-layer and three-layer specimens respectively in terms of the elastic modulus of the ‘cladding’ layer 1 normalized by that of layer 2 \( (E_2 = 200 \text{ GPa}) \). Curves for different values of the temperature gradient exponent \( n \) (equation (1)) are shown. In all cases, the initial furnace temperature was kept at \( T_0 = 950^\circ \text{C} \) and the maximum crack depth at \( a = 50 \mu \text{m} \). Note also that the calibrated \( F \) versus \( E_1/E_2 \) curves were found to be unaffected by the value of the temperature differential \( \Delta T \), for \( \Delta T = 100–200^\circ \text{C} \). The results show variations in \( F \) of approximately ±15\% for a wide range of \( E_1/E_2 \) ratios. The effects of the maximum depth of the subclad interfacial crack on the calibrated \( F \) values for the two-layer and three-layer specimens are given in figures 10(a) and (b) respectively. It can be seen that the results for the two-layer structure are more sensitive to the value of \( a \) but less to that of the temperature gradient exponent \( n \). Note that, in the limiting case of a single layer, \( F = 0 \), and the subcrack problem in the multilayer system simplifies to that of a surface crack in the YSZ layer.

The implications of increasing the elastic stiffness of the cladding layer on the crack’s maximum SIF can be inferred from figure 11, which shows both the SIFs obtained analytically from equation (8) and the corresponding numerical values extracted from the linear elastic finite-element analyses of the cracked systems for different \( \Delta T \) values. Here, \( a = 0.25h_2 \) and \( n = 0.5 \). It can be seen that the agreement is accurate throughout the range of conditions considered. It is also interesting to note that an increase in the \( E_1/E_2 \) ratio from 0.25, which corresponds to the material systems tested in this work, to 1 results in a decrease in \( K_1 \) owing to the higher stress which develops in the cladding layer 1. Conversely, a decrease in the \( E_1/E_2 \) ratio is associated with an increase in \( K_1 \). The least conservative condition is obtained in the limiting case of \( E_1/E_2 = 0 \), where the solution for an edge through-crack on either a plate or a bimaterial system is recovered.

The applicability of the proposed failure prediction methodology to the one-, two- and three-layer structures investigated in this work is based on the understand-
ing that the failure of all tested specimens was caused by the propagation of the same types of defect. Thus, a confirmation that all the fracture-controlling defects indeed come from the same population will require that the range of critical predicted crack depths in all the tested specimens using equations (5)–(8) be accurately described by a single statistical distribution function. Here, a Weibull function was found to represent best the statistical distribution of critical crack depths. Then the probability of finding a crack of size $a_C$ within a material volume $V$ is given as,

$$ P_a = 1 - \exp \left[ - \int_V \left( \frac{a_C}{a_w} \right)^{m_C} \, dV \right], \quad (9) $$

where $a_w$ and $m_C$ are the Weibull scaling parameter and modulus respectively.

The resulting Weibull plot showing the range of $a_C$ values extracted from the weight function solutions (equations (5)–(8)) using the thermal stresses at failure for each of the thermally shocked specimens and a given fracture toughness is given in

Figure 9. Variation in the coefficient $F$ with the ratio $E_1/E_2$ for (a) the two-layer and (a) the three-layer structures. Results were found to be independent of $\Delta T$, for $\Delta T = 100–200 ^\circ C$. 
figure 12. Here, the critical crack depths were calculated using fracture toughness values for the YSZ consistent with published values, namely $K_{IC} = 1.0$, $1.3$ and $1.6$ MPa m$^{1/2}$. Minh (1993) reported fracture toughness values for fully stabilized 8% YSZ at room temperature within the 1–3 MPa m$^{1/2}$ range. However, recent work reported values of fracture toughness for tape-cast YSZ, used as the electrolyte material in SOFCs, at the lower end of this range (Selçuk and Atkinson 2000). Using indentation and double-torsion loading techniques, fracture toughnesses of $1.61 \pm 0.12$ MPa m$^{1/2}$ at room temperature and $1.01 \pm 0.05$ MPa m$^{1/2}$ at $900^\circ$C were measured by Selçuk and Atkinson (2000). It should also be pointed out that the value of $K_{IC}$, extracted from those tested specimens where an accurate identification of the initial defect size was made, gave $K_{IC} = 1.1$ MPa m$^{1/2}$, which is within the range of values reported by Selçuk and Atkinson (2000).

The results shown in figure 12 reveal that all the test data can be described adequately by single statistical distribution law, irrespective of the assumed YSZ fracture toughness. Moreover, a single value of the Weibull modulus, $m_C = 4.5$
(given by the slope of the solid lines in figure 12), seems to reproduce the overall trend exhibited by the data well, irrespective of the assumed $K_{IC}$. From these observations, it can therefore be concluded that all the defects from which thermal shock failure initiated are of the same type.

The proposed analytical approach has been used to construct regions of fail-safe design for the two- and three-layer ceramic structures. The effects of the temperature gradient exponent on the critical values of $\Delta T$ required to propagate pre-existing defects of different sizes in the two- and three-layer systems are given in figures 13 (a) and (b) respectively. Here, the upper and lower bounds correspond to the maximum

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**Figure 11.** Comparison between the analytically estimated $K_I$ for the interfacial crack in a three-layer structure and the finite-element (FE) results for different temperature differentials $\Delta T$ and $E_1/E_2$ ratios.

**Figure 12.** Weibull distribution of critical subclad interfacial crack lengths in the one-layer (1L), two-layer (2L) and three-layer (3L) systems for different values of YSZ fracture toughness. Here, $E_1/E_2 = E_3/E_2 = 0.25$. 
and minimum critical crack depths extracted from the known range of calculated failure stresses in the thermally shocked specimens. A value of $K_{IC} = 1 \text{ MPa m}^{1/2}$ was used in the analysis to obtain conservative estimates. With this information, it is possible to select, for a given value of the temperature gradient exponent $n$ (or $\Delta T$), the maximum value of $\Delta T$ (or $n$) which the two- and three-layer systems with different defect sizes could tolerate without failing. These results highlight the importance of limiting the size of the maximum allowable surface defects in the YSZ to a level consistent with the expected thermal loads.
§ 6. Conclusions

The thermally induced fracture behaviour of multilayer ceramic structures has been investigated experimentally and a fracture-mechanics-based failure assessment methodology proposed. The general fracture patterns and the dominant fracture mode, namely unstable crack propagation from subclad interfacial cracks, have been identified. A generic analytical weight-function-based solution has been presented which allows the SIFs of subrack interfacial cracks in two- and three-layer ceramic structures subjected to inhomogeneous in-plane temperature distributions to be determined. Failure diagrams have been constructed to determine fail-safe thermal transient conditions for multilayer ceramic structures used in SOFCs and other technologies.

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Appendix A

Weight function

An analytical expression for the weight function \( m(\bar{z}, \bar{a}) \) used in equation (4) was obtained by fitting the \( m(\bar{z}, \bar{a}) \) values for planar structures given in graphical form by Labbens et al. (1976) with a maximum error of \( \pm 0.5\% \). Here,

\[
m(\bar{z}, \bar{a}) = f_1(\bar{z}) + f_2(\bar{z})\bar{a} + f_3(\bar{z})\bar{a}^2,
\]

where \( \bar{z} \) is the distance from the cracked surface, \( \bar{z} = z/w, \bar{a} = a/w \) is the normalized crack depth, and \( f_1, f_2 \) and \( f_3 \) are parameters given in table A1 in terms of \( \bar{z} \).

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