Elastoviscoplasticity at finite strain

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Plan

1. Objective derivation, local objective frames

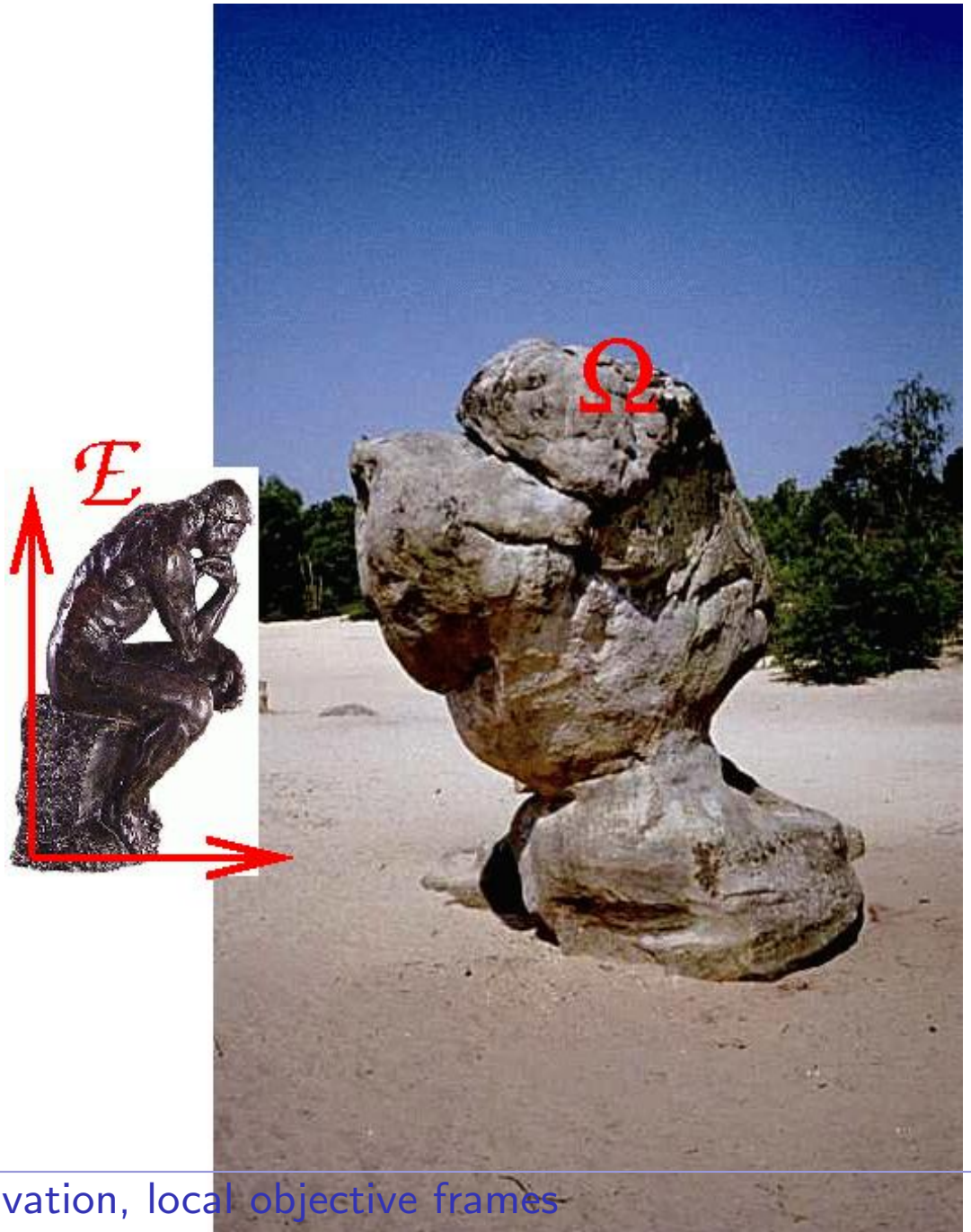
2. Finite deformation elastoviscoplasticity
   - Elasticity, Hypoelasticity, Hyperelasticity
   - Multiplicative decomposition
   - Formulation in objective local frames
   - Example: simple glide

3. Conclusions et recommandations

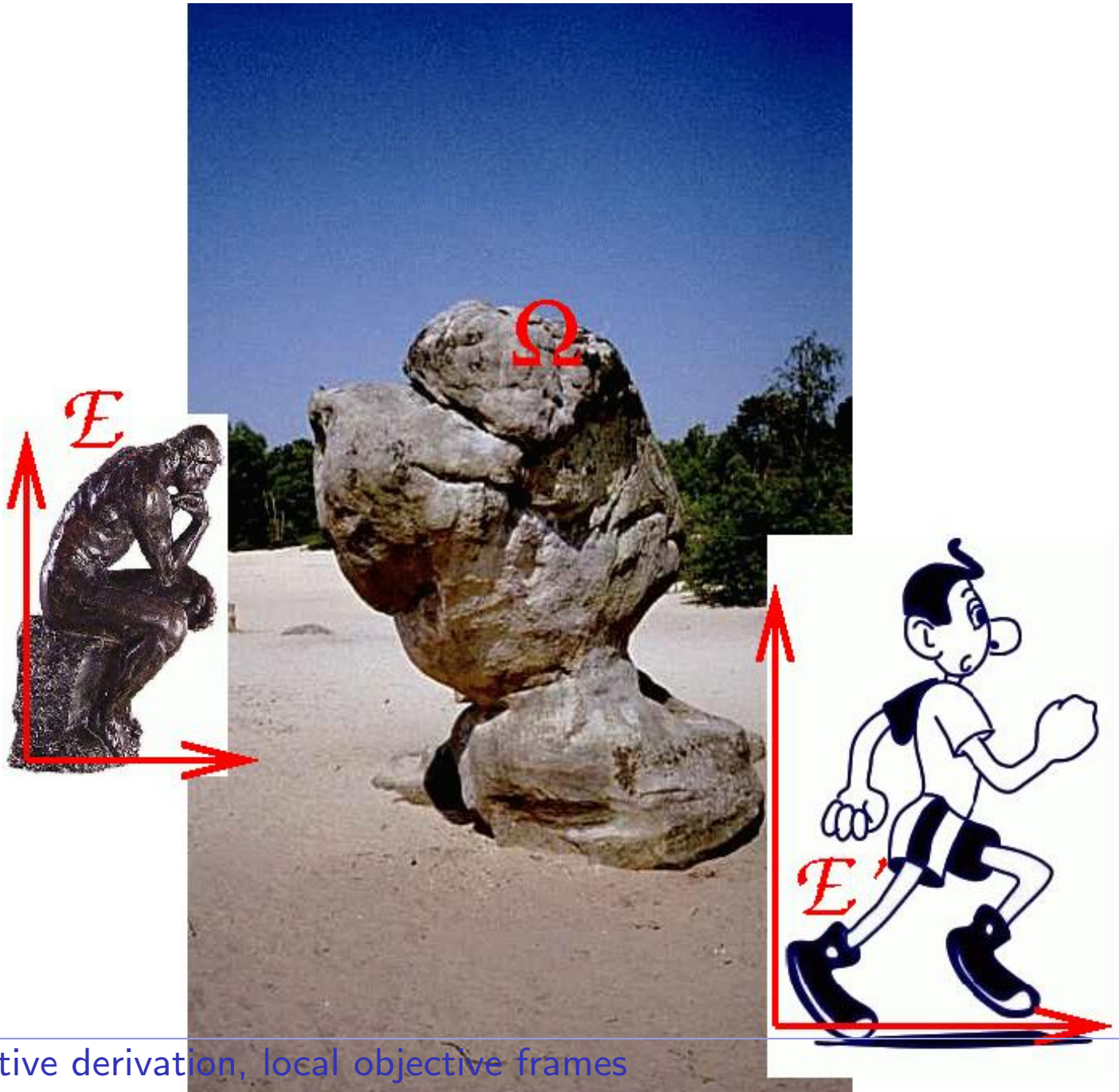
Plan

Space and observers

Space and observers



Space and observers



Space and observers

- Change of observer :
  observer $(\mathcal{E}, E)$, space point $\underline{\mathbf{x}}$, another observer $(\mathcal{E}', E')$
  Galilean/Euclidean transformations (trafo)

$$\underline{\mathbf{x}}' = \underset{\sim}{\mathbf{Q}}(t).\underline{\mathbf{x}} + \underline{\mathbf{c}}(t)$$

- objective quantities
  scalars

$$m' = m$$

  vectors

$$\underline{\mathbf{u}}' = \underset{\sim}{\mathbf{Q}}(t).\underline{\mathbf{u}}$$

  tensors

$$\underset{\sim}{\mathbf{T}} = \underline{\mathbf{u}} \otimes \underline{\mathbf{v}}$$

$$\underset{\sim}{\mathbf{T}}' = \underline{\mathbf{u}}' \otimes \underline{\mathbf{v}}' = \underset{\sim}{\mathbf{Q}}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{Q}}^T$$

- invariant quantities

$$m' = m, \quad \underset{\sim}{\mathbf{T}}' = \underset{\sim}{\mathbf{T}}$$

Transport rules

$$\tau_{F^{-1}F}(\underset{\sim}{\mathbf{T}}) = \underset{\sim}{\mathbf{F}}^{-1}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{F}}$$

$$\tau_{F^{-1}F^{-T}}(\underset{\sim}{\mathbf{T}}) = \underset{\sim}{\mathbf{F}}^{-1}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{F}}^{-T}$$

$$\tau_{F^{T}F^{-T}}(\underset{\sim}{\mathbf{T}}) = \underset{\sim}{\mathbf{F}}^{T}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{F}}^{-T}$$

$$\tau_{F^{T}F}(\underset{\sim}{\mathbf{T}}) = \underset{\sim}{\mathbf{F}}^{T}.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{F}}.$$

invariariance of the inner product of pulled-back tensors

$$\underset{\sim}{\mathbf{A}} : \underset{\sim}{\mathbf{B}} = \tau_{F^{-1}F}(\underset{\sim}{\mathbf{A}}) : \tau_{F^{T}F^{-T}}(\underset{\sim}{\mathbf{B}}) = \tau_{F^{T}F}(\underset{\sim}{\mathbf{A}}) : \tau_{F^{-1}F^{-T}}(\underset{\sim}{\mathbf{B}}).$$

Objective derivation

time derivative of a rigid vector followed by a moving observer

$$\dot{\underline{\mathbf{u}}}' = \dot{\underset{\sim}{\mathbf{Q}}}.\underset{\sim}{\mathbf{Q}}^T.\underline{\mathbf{u}}'$$

it is non zero!

Define then

$$D^J\underline{\mathbf{u}}' = \dot{\underline{\mathbf{u}}}' - \underset{\sim}{\mathbf{W}}.\underline{\mathbf{u}}'$$

with the spin of the continuum $\underset{\sim}{\mathbf{W}} = \dot{\underset{\sim}{\mathbf{Q}}}.\underset{\sim}{\mathbf{Q}}^T$

for $\underset{\sim}{\mathbf{T}} = \underline{\mathbf{u}} \otimes \underline{\mathbf{v}}$, one defines

$$\underset{\sim}{\mathbf{T}}^J = \underline{\mathbf{u}}^J \otimes \underline{\mathbf{v}} + \underline{\mathbf{u}} \otimes \underline{\mathbf{v}}^J$$

hence

$$\underset{\sim}{\mathbf{T}}^J = \dot{\underset{\sim}{\mathbf{T}}} + \underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{W}} - \underset{\sim}{\mathbf{W}}.\underset{\sim}{\mathbf{T}}$$

called Jaumann derivative

Converctive derivatives

push forward–time derivative–pull back

$$\underset{\sim}{\mathbf{T}}^{(1)} = \underset{\sim}{\mathbf{F}}(\underset{\sim}{\mathbf{F}}^{-1}\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{F}})^{\cdot}\underset{\sim}{\mathbf{F}}^{-1} = \dot{\underset{\sim}{\mathbf{T}}} - \underset{\sim}{\mathbf{L}}\underset{\sim}{\mathbf{T}} + \underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{L}}$$

$$\underset{\sim}{\mathbf{T}}^{(2)} = \underset{\sim}{\mathbf{F}}(\underset{\sim}{\mathbf{F}}^{-1}\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{F}}^{-T})^{\cdot}\underset{\sim}{\mathbf{F}}^{T} = \dot{\underset{\sim}{\mathbf{T}}} - \underset{\sim}{\mathbf{L}}\underset{\sim}{\mathbf{T}} - \underset{\sim}{\mathbf{L}}^{T}\underset{\sim}{\mathbf{T}}$$

$$\underset{\sim}{\mathbf{T}}^{(3)} = \underset{\sim}{\mathbf{F}}^{-T}(\underset{\sim}{\mathbf{F}}^{T}\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{F}}^{-T})^{\cdot}\underset{\sim}{\mathbf{F}}^{T} = \dot{\underset{\sim}{\mathbf{T}}} + \underset{\sim}{\mathbf{L}}^{T}\underset{\sim}{\mathbf{T}} - \underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{L}}^{T}$$

$$\underset{\sim}{\mathbf{T}}^{(4)} = \underset{\sim}{\mathbf{F}}^{-T}(\underset{\sim}{\mathbf{F}}^{T}\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{F}})^{\cdot}\underset{\sim}{\mathbf{F}}^{-1} = \dot{\underset{\sim}{\mathbf{T}}} + \underset{\sim}{\mathbf{L}}^{T}\underset{\sim}{\mathbf{T}} + \underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{L}}.$$

Note that

$$\underset{\sim}{\mathbf{T}}^{J} = \frac{1}{2}(\underset{\sim}{\mathbf{T}}^{(1)} + \underset{\sim}{\mathbf{T}}^{(3)})$$

$$\tau_{JF^{-1}F^{-T}}(\underset{\sim}{\mathbf{T}}) = J\underset{\sim}{\mathbf{F}}^{-1}\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{F}}^{-T}$$

the associated objective derivative

$$\underset{\sim}{\mathbf{T}}^{(5)} = \frac{1}{J}\underset{\sim}{\mathbf{F}}(\tau_{JF^{-1}F^{-T}}(\underset{\sim}{\mathbf{T}}))^{\bullet}\underset{\sim}{\mathbf{F}}^{T} = \dot{\underset{\sim}{\mathbf{T}}} - \underset{\sim}{\mathbf{L}}\underset{\sim}{\mathbf{T}} - \underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{L}}^{T} + \underset{\sim}{\mathbf{T}}\,\mathrm{Tr}\,\underset{\sim}{\mathbf{L}}$$

is called Truesdells derivative.

Derivation in an objective local frame

objective local frame: family of observers $\mathcal{E}_{\underline{\mathbf{x}}}$ where $\mathcal{E}_{\underline{\mathbf{x}}}$ is the priviledged local frame at $\underline{\mathbf{x}}$, and $\underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}}$ the associated rotation. Local frame are objective if

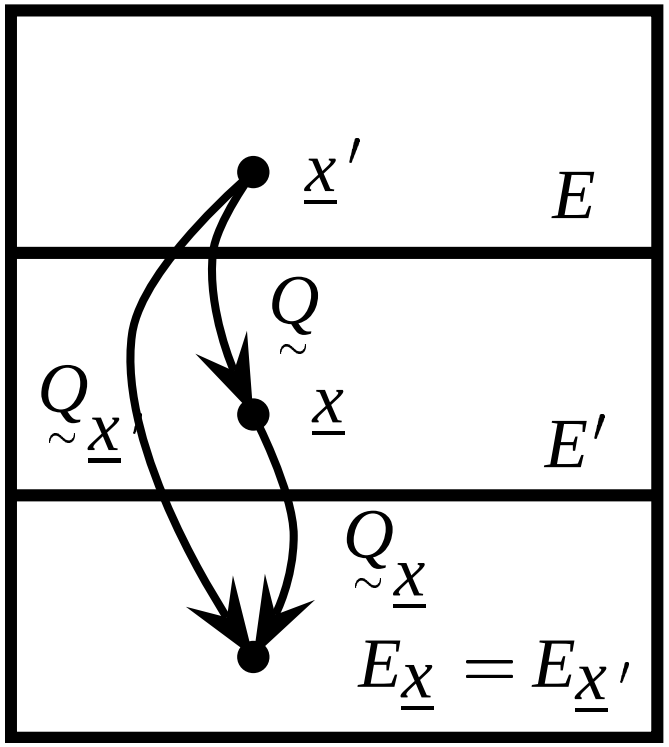
$$\underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}'} = \underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}} \underset{\sim}{\mathbf{Q}}^T$$

time derivative of $\underset{\sim}{\mathbf{T}}$ in $\mathcal{E}_{\underline{\mathbf{x}}}$:

$$D_{\mathcal{E},\underline{\mathbf{x}}} \underset{\sim}{\mathbf{T}} = \underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}}^T \overbrace{(\underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}} \underset{\sim}{\mathbf{T}} \underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}}^T)}^{\bullet} \underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}}$$

$$D_{\mathcal{E}} \underset{\sim}{\mathbf{T}} = \dot{\underset{\sim}{\mathbf{T}}} + \underset{\sim}{\mathbf{T}} \underset{\sim}{\mathbf{\Omega}}_{\mathcal{E}_I} - \underset{\sim}{\mathbf{\Omega}}_{\mathcal{E}_I} \underset{\sim}{\mathbf{T}}$$

where $\underset{\sim}{\mathbf{\Omega}}_{\mathcal{E}_I} = \dot{\underset{\sim}{\mathbf{Q}}}_{\underline{\mathbf{x}}}^T \underset{\sim}{\mathbf{Q}}_{\underline{\mathbf{x}}}$



Corotational and polar observers

- Corotational frame: there is a unique family of objective local frames $\mathcal{E}^c_{\underline{\mathbf{x}}}$ such that, at each material point and each time, the spin tensor with respect to this observer vanishes;

$$\forall \underline{\mathbf{x}} \in \Omega, \quad \underset{\sim}{\mathbf{W}}' = \underset{\sim}{\dot{\mathbf{Q}}}.\underset{\sim}{\mathbf{Q}}^T + \underset{\sim}{\mathbf{Q}}.\underset{\sim}{\mathbf{W}}.\underset{\sim}{\mathbf{Q}}^T$$

in order to have $\underset{\sim}{\mathbf{W}}' = 0$, one must have

$$-\underset{\sim}{\mathbf{Q}}_c^T.\underset{\sim}{\dot{\mathbf{Q}}}_c = \underset{\sim}{\dot{\mathbf{Q}}}_c^T.\underset{\sim}{\mathbf{Q}}_c = \underset{\sim}{\mathbf{W}} \; (\underset{\sim}{\mathbf{Q}}_{c\underline{\mathbf{x}}}(t) = \underset{\sim}{\mathbf{Q}}_c(\underline{\mathbf{x}}, t))$$

the corresponding objective derivative is

$$D_{\mathcal{E}^c}\underset{\sim}{\mathbf{T}} = \underset{\sim}{\mathbf{Q}}_c^T.(\underset{\sim}{\mathbf{Q}}_c.\underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{Q}}_c^T)^{\cdot}\underset{\sim}{\mathbf{Q}}_c = \underset{\sim}{\dot{\mathbf{T}}} + \underset{\sim}{\mathbf{T}}.\underset{\sim}{\mathbf{W}} - \underset{\sim}{\mathbf{W}}.\underset{\sim}{\mathbf{T}}$$

i.e. Jaumann derivative!

- Polar frame : polaire decomposition

$$\underset{\sim}{\mathbf{F}} = \underset{\sim}{\mathbf{R}}.\underset{\sim}{\mathbf{U}} = \underset{\sim}{\mathbf{V}}.\underset{\sim}{\mathbf{R}}$$

$$\underset{\sim}{\mathbf{Q}}_{\mathcal{E}_R} = \underset{\sim}{\mathbf{R}}^T$$

this frame depends on the reference configuration

A few unacceptable constitutive laws...

Why are the following illicit constitutive relations?

$$\underset{\sim}{\boldsymbol{\sigma}} = \eta \underline{\mathbf{x}} \otimes \underline{\mathbf{x}}$$

$$\underset{\sim}{\boldsymbol{\sigma}} = \eta \underline{\mathbf{v}} \otimes \underline{\mathbf{v}}$$

$$\underset{\sim}{\mathbf{B}} = \eta t^n \underset{\sim}{\boldsymbol{\sigma}}$$

$$\dot{\underset{\sim}{\boldsymbol{\sigma}}} = \eta \underset{\sim}{\mathbf{D}}$$

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Hyperelasticity

$$\Phi^i = (\underset{\sim}{\mathbf{\Pi}} - \rho \frac{\partial \Psi}{\partial \underset{\sim}{\mathbf{E}}}) : \dot{\underset{\sim}{\mathbf{E}}} - \rho(s + \frac{\partial \Psi}{\partial T})\dot{T} \geq 0$$

$$\begin{aligned} \underset{\sim}{\mathbf{\Pi}} &= \rho_0 \frac{\partial \psi_0}{\partial \underset{\sim}{\mathbf{E}}} \\ \eta_0 &= -\frac{\partial \psi_0}{\partial T} \\ \frac{\partial \psi_0}{\partial \underline{\mathbf{G}}} &= 0 \end{aligned}$$

other strain measures can be used. Eulerian form:

$$\underset{\sim}{\boldsymbol{\sigma}} = 2\rho \frac{\partial \Psi}{\partial \underset{\sim}{\mathbf{B}}} . \underset{\sim}{\mathbf{B}}$$

(anisotropy...)

Example: elastic law which is not hyperelastic

lasticite : strain dependence, no dissipation, reversible

$$W(\underset{\sim}{\mathbf{E}}_A \to \underset{\sim}{\mathbf{E}}_B) = \int_{t_A}^{t_B} \rho_0 \frac{\partial \psi}{\partial \underset{\sim}{\mathbf{E}}} : \dot{\underset{\sim}{\mathbf{E}}}\, dt = \left[\rho_0 \psi_0(\underset{\sim}{\mathbf{E}}, T)\right]_A^B = \rho_0 \psi_0(\underset{\sim}{\mathbf{E}}_B, T_B) - \rho_0 \psi_0(\underset{\sim}{\mathbf{E}}_A,$$

This elastic law is not hyperelastic

$$\underset{\sim}{\mathbf{\Pi}} = \frac{\alpha}{2}(\mathrm{trace}\, \underset{\sim}{\mathbf{E}}^2)\, \underset{\sim}{\mathbf{1}}, \quad \frac{\partial \alpha_0}{\partial I_2} = 2 \neq \frac{\partial \alpha_1}{\partial I_1} = 0$$

to see it, consider two strain paths, with $\lambda : 0 \longrightarrow 1$:

$$[\underset{\sim}{\mathbf{E}}_1(\lambda)] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [\underset{\sim}{\mathbf{E}}_2(\lambda)] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$W_1 = \int_{t_A}^{t_B} 2\alpha\lambda\dot{\lambda}\, dt = \frac{2}{3}\alpha, \quad W_2 = \int_{t_A}^{t_B} \frac{\alpha}{2}(\lambda^2 + \lambda^4)(1 + 2\lambda)\dot{\lambda}\, dt = \frac{41}{60}\alpha$$

The work done depends on the strain path...

Hypoelasticity

$$\overset{\nabla}{\underset{\sim}{\boldsymbol{\sigma}}} = f(\underset{\sim}{\mathbf{D}})$$

where $\underset{\sim}{\boldsymbol{\sigma}}$ is an objective derivative.
major defect: generally not hyperelastic, with the known consequences...

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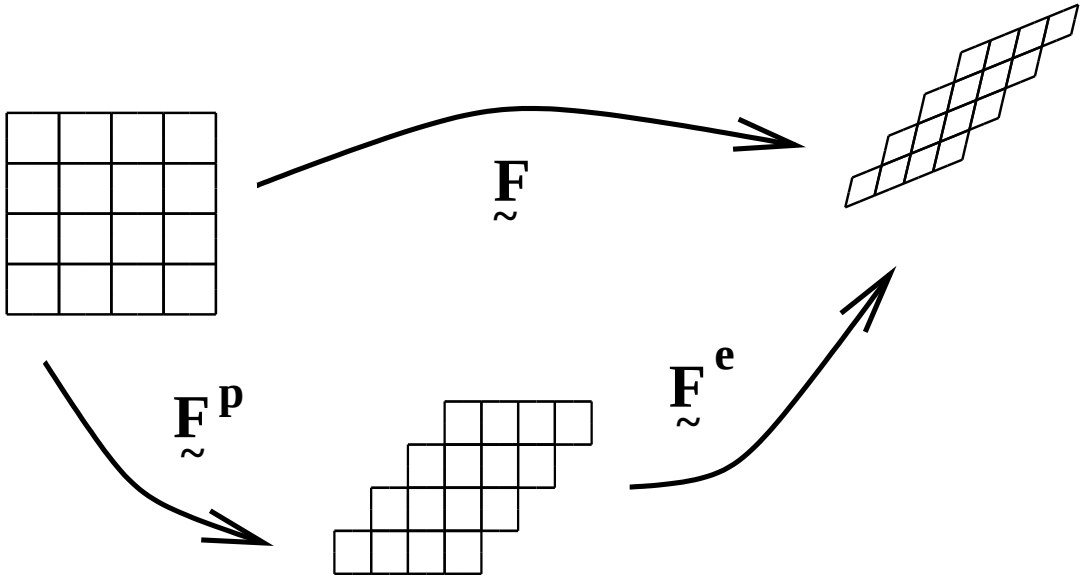
Anisotropic plasticity at finite strain

existence of **a triad of directors**



$$\underset{\sim}{\mathbf{F}} = \underset{\sim}{\mathbf{F}}^e . \underset{\sim}{\mathbf{F}}^p$$

intermediate stress-released configuration **isoclinic** (local, without physical reality (kinematic hardening, viscoplasticity))

archtype : le monocristal [Mandel 1973]

the rotation of the directors is defined by the *plastic spin* : this is an additional constitutive law.

Standard generalized materials (1)

- elastic deformation and stress tensor pulled–back to the intermediate configuration:

$$\underset{\sim}{\mathbf{E}}^e = \frac{1}{2}(\underset{\sim}{\mathbf{F}}^e.\underset{\sim}{\mathbf{F}}^{eT} - \underset{\sim}{\mathbf{1}}), \quad \underset{\sim}{\mathbf{\Pi}}^e = \frac{\rho_i}{\rho}\underset{\sim}{\mathbf{F}}^{e-1}.\underset{\sim}{\boldsymbol{\sigma}}.\underset{\sim}{\mathbf{F}}^{e-T}$$

- work of internal forces

$$\frac{1}{\rho}\underset{\sim}{\boldsymbol{\sigma}} : (\underset{\sim}{\dot{\mathbf{F}}}.\underset{\sim}{\mathbf{F}}^{-1}) = \frac{1}{\rho_i}(\underset{\sim}{\mathbf{\Pi}}^e : \underset{\sim}{\dot{\mathbf{E}}}^e + (\underset{\sim}{\mathbf{F}}^{et}.\underset{\sim}{\mathbf{F}}^e.\underset{\sim}{\mathbf{\Pi}}^e) : (\underset{\sim}{\dot{\mathbf{F}}}^p.\underset{\sim}{\mathbf{F}}^{p-1}))$$

- Clausius-Duhem inequality:

$$\rho(\frac{\underset{\sim}{\mathbf{\Pi}}^e}{\rho_i} - \frac{\partial\Psi}{\partial\underset{\sim}{\mathbf{E}}^e}) : \underset{\sim}{\dot{\mathbf{E}}}^e - \rho(s + \frac{\partial\Psi}{\partial T})\dot{T} - \rho\frac{\partial\Psi}{\partial\underset{\sim}{\boldsymbol{\alpha}}} : \underset{\sim}{\dot{\boldsymbol{\alpha}}} \geq 0$$

state laws:

$$\frac{\underset{\sim}{\mathbf{\Pi}}^e}{\rho_i} = \frac{\partial\Psi}{\partial\underset{\sim}{\mathbf{E}}^e}, \quad \underset{\sim}{\mathbf{X}} = \rho\frac{\partial\Psi}{\partial\underset{\sim}{\boldsymbol{\alpha}}}, \quad s = -\frac{\partial\Psi}{\partial T}$$

Standard generalized materials (2)

- intrinsic dissipation : $D = \underset{\sim}{\mathbf{M}} : \underset{\sim}{\dot{\mathbf{F}}}^p . \underset{\sim}{\mathbf{F}}^{p-1} - \underset{\sim}{\mathbf{X}} : \underset{\sim}{\dot{\boldsymbol{\alpha}}}$
  Mandel stress tensor:

$$\underset{\sim}{\mathbf{M}} := \underset{\sim}{\mathbf{F}}^{eT} . \underset{\sim}{\mathbf{F}}^{e} . \frac{\underset{\sim}{\boldsymbol{\Pi}}^{e}}{\rho_i}$$

- dissipation potential

$$\underset{\sim}{\dot{\mathbf{F}}}^p . \underset{\sim}{\mathbf{F}}^{p-1} = \frac{\partial \Omega}{\partial \underset{\sim}{\mathbf{M}}}, \quad \underset{\sim}{\dot{\boldsymbol{\alpha}}} = \frac{\partial \Omega}{\partial \underset{\sim}{\mathbf{X}}}$$

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Formulating constitutive equations in objective local frames

Take a constitutive model of the form

$$\dot{\underset{\sim}{\boldsymbol{\sigma}}}^* = f(\underset{\sim}{\mathbf{D}}^*)$$

where $\underset{\sim}{\boldsymbol{\sigma}}^* = \underset{\sim}{\mathbf{Q}}^* \underset{\sim}{\boldsymbol{\sigma}} \underset{\sim}{\mathbf{Q}}^{*T}$ et $\underset{\sim}{\mathbf{D}}^* = \underset{\sim}{\mathbf{Q}}^* \underset{\sim}{\mathbf{D}} \underset{\sim}{\mathbf{Q}}^{*T}$ are the stress tensor and the strain rate pushed forward in an objective local frame.

Example : corotational frame, isotropic case

$$\dot{\underset{\sim}{\boldsymbol{\sigma}}}^c = \lambda(\mathrm{Tr}\, \underset{\sim}{\mathbf{D}}^c)\underset{\sim}{\mathbf{1}} + 2\mu \underset{\sim}{\mathbf{D}}^c$$

and pull it back again in the current frame:

$$\underset{\sim}{\mathbf{Q}}^{cT} \dot{\underset{\sim}{\boldsymbol{\sigma}}}^c \underset{\sim}{\mathbf{Q}}^c = \underset{\sim}{\boldsymbol{\sigma}}^J = \lambda(\mathrm{Tr}\, \underset{\sim}{\mathbf{D}})\underset{\sim}{\mathbf{1}} + 2\mu \underset{\sim}{\mathbf{D}}$$

where the Jaumann derivative comes in.

advantage : f arbitrary (anisotropic)... inconvenient: generally hypoelastic...

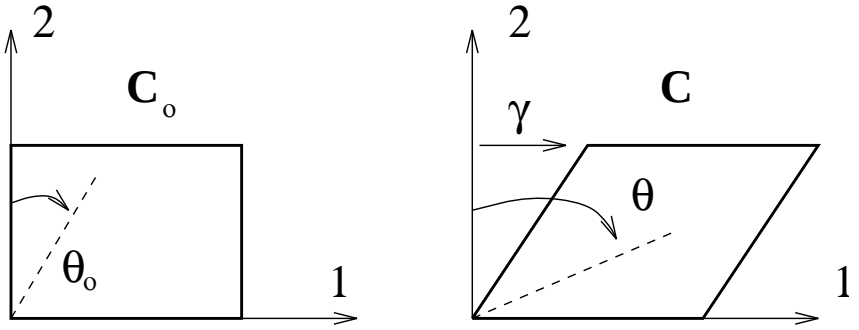
Elastoviscoplastic constitutive laws in objective local frames

$$\begin{cases} \dot{\underset{\sim}{\varepsilon}} = \dot{\underset{\sim}{\varepsilon}}^e + \dot{\underset{\sim}{\varepsilon}}^p \\ \dot{\underset{\sim}{\varepsilon}}^p = f(\underset{\sim}{\boldsymbol{\sigma}}, \underset{\sim}{\boldsymbol{\alpha}}) \\ \underset{\sim}{\boldsymbol{\sigma}} = \underset{\sim}{\mathbf{C}} \ : \underset{\sim}{\varepsilon}^e \\ \dot{\underset{\sim}{\boldsymbol{\alpha}}} = h(\alpha, \dot{\underset{\sim}{\varepsilon}}^p) \end{cases} \Longrightarrow \begin{array}{c} \dot{\underset{\sim}{\mathbf{e}}} = \underset{\sim}{\mathbf{Q}}^T.\underset{\sim}{\mathbf{D}}.\underset{\sim}{\mathbf{Q}}, \\ \underset{\sim}{\mathbf{s}} = \underset{\sim}{\mathbf{Q}}^T.\frac{\rho_0}{\rho}\underset{\sim}{\boldsymbol{\sigma}}.\underset{\sim}{\mathbf{Q}} \end{array} \Longrightarrow \begin{cases} \dot{\underset{\sim}{\mathbf{e}}} = \dot{\underset{\sim}{\mathbf{e}}}^e + \dot{\underset{\sim}{\mathbf{e}}}^p \\ \dot{\underset{\sim}{\mathbf{e}}}^p = f(\underset{\sim}{\mathbf{s}}, \underset{\sim}{\boldsymbol{\alpha}}) \\ \underset{\sim}{\mathbf{s}} = \underset{\sim}{\mathbf{C}} \ : \underset{\sim}{\mathbf{e}}^e \\ \dot{\underset{\sim}{\boldsymbol{\alpha}}} = h(\underset{\sim}{\boldsymbol{\alpha}}, \dot{\underset{\sim}{\mathbf{e}}}^p) \end{cases}$$

- corotational frame: $\underset{\sim}{\mathbf{Q}}$ tel que $\dot{\underset{\sim}{\mathbf{Q}}}^T.\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{W}}$
- polar frame: $\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{R}}$

Plan

Simple glide



$$\underset{\sim}{\mathbf{F}} = \underset{\sim}{\mathbf{1}} + \gamma \underline{\mathbf{e}}_1 \otimes \underline{\mathbf{e}}_2$$

rotation of material line elements:

$$\tan\theta = \tan\theta_0 \ + \ \gamma, \quad \dot{\theta} \ = \ \frac{\dot{\gamma}}{1 \ + \ (\tan\theta_0 \ + \ \gamma)^2}$$

$$< \dot{\theta}_L >= \frac{\dot{\gamma}}{\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{1 \ + \ (\tan\theta_0 \ + \ \gamma)^2} \, d\theta_0 \ = \ \frac{\dot{\gamma}}{2(1 + \frac{\gamma^2}{4})}$$

$$< \dot{\theta}_e >= \frac{\dot{\gamma}}{\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{1 \ + \ \tan^2\theta} \, d\theta \ = \ \frac{\dot{\gamma}}{2}$$

endless rotation of the corotation frame... saturation of the polar frame...

Simple glide and elasticity

- Lagrangean formulation

$$\underset{\sim}{\mathbf{\Pi}} = 2\mu \, \underset{\sim}{\mathbf{E}} + \lambda \mathrm{Tr} \, \underset{\sim}{\mathbf{E}} \, \underset{\sim}{\mathbf{1}}$$

  $\underset{\sim}{\mathbf{\Pi}}$ et $\underset{\sim}{\mathbf{E}}$ respectively are the second Piola-Kirchhoff stress tensor and the Green–Lagrange strain tensor.

- Eulerian formulation

$$\underset{\sim}{\boldsymbol{\sigma}} = 2\mu \, \log \underset{\sim}{\mathbf{V}} + \lambda \, \mathrm{Tr} \, (\log \underset{\sim}{\mathbf{V}}) \, \underset{\sim}{\mathbf{1}}$$

  $\underset{\sim}{\mathbf{V}}$ comes from the polar decomposition $\underset{\sim}{\mathbf{F}} = \underset{\sim}{\mathbf{V}} \underset{\sim}{\mathbf{R}}$

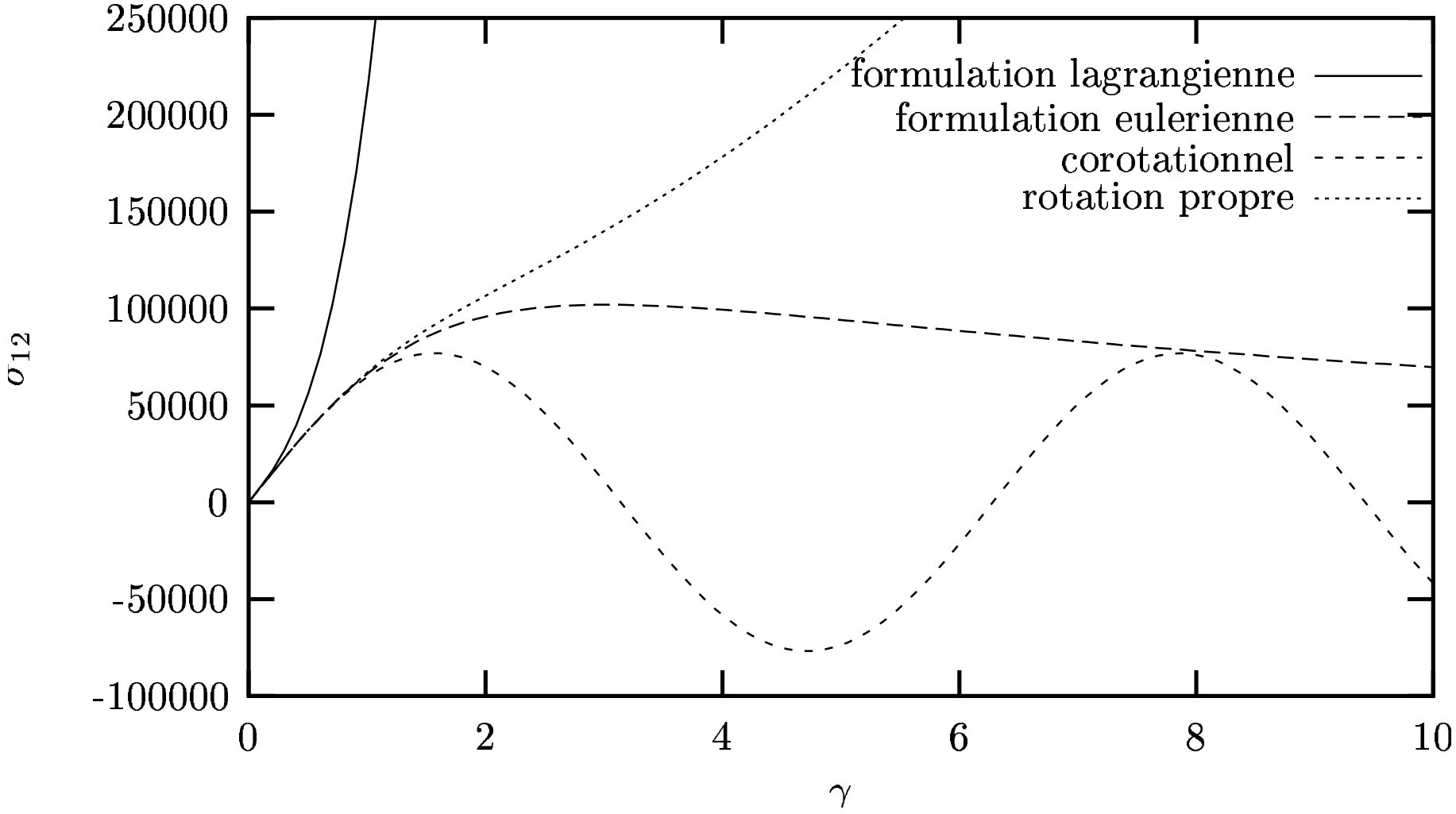
- formulation based on objective local frames

$$\underset{\sim}{\mathbf{s}} = 2\mu \, \underset{\sim}{\mathbf{e}} + \lambda \mathrm{Tr} \, \underset{\sim}{\mathbf{e}} \, \underset{\sim}{\mathbf{1}}$$

$$\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{Q}}_c \quad (\text{corotational})$$

$$\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{R}} \quad (\text{polarrotation})$$

Simple glide and elasticity



Simple glide in plasticity

rigid plastic case using the corotational frame :

- criterion $$f(\underset{\sim}{\mathbf{s}}, R, \underset{\sim}{\mathbf{X}}) = J_2(\underset{\sim}{\mathbf{s}} - \underset{\sim}{\mathbf{X}}) - R$$

$$\dot{p} = \sqrt{\frac{2}{3}\,\underset{\sim}{\mathbf{D}} : \underset{\sim}{\mathbf{D}}} \quad J_2(\underset{\sim}{\mathbf{s}} - \underset{\sim}{\mathbf{X}}) = \sqrt{\frac{3}{2}\,(\underset{\sim}{\mathbf{s}}^{dev} - \underset{\sim}{\mathbf{X}}) : (\underset{\sim}{\mathbf{s}}^{dev} - \underset{\sim}{\mathbf{X}})}$$

- flow rule $$\dot{\underset{\sim}{\mathbf{e}}} = \dot{p}\,\frac{\partial f}{\partial \underset{\sim}{\mathbf{s}}}$$

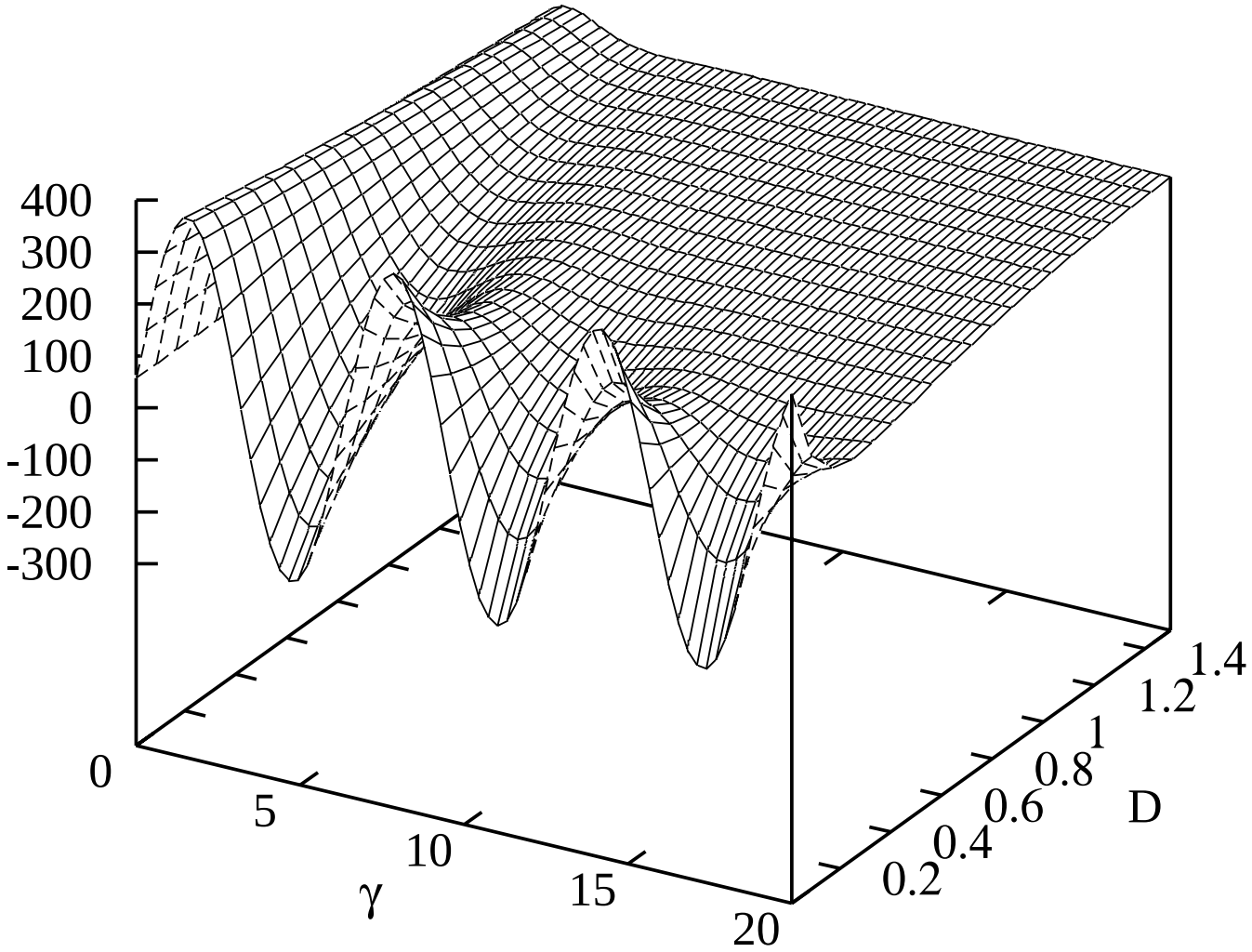
- evolution law for kinematic hardening

$$\dot{\underset{\sim}{\mathbf{X}}} = \frac{2}{3}\,C\,\dot{\underset{\sim}{\mathbf{e}}}^p \;-\; D\,\dot{p}\,\underset{\sim}{\mathbf{X}}$$

- solution:

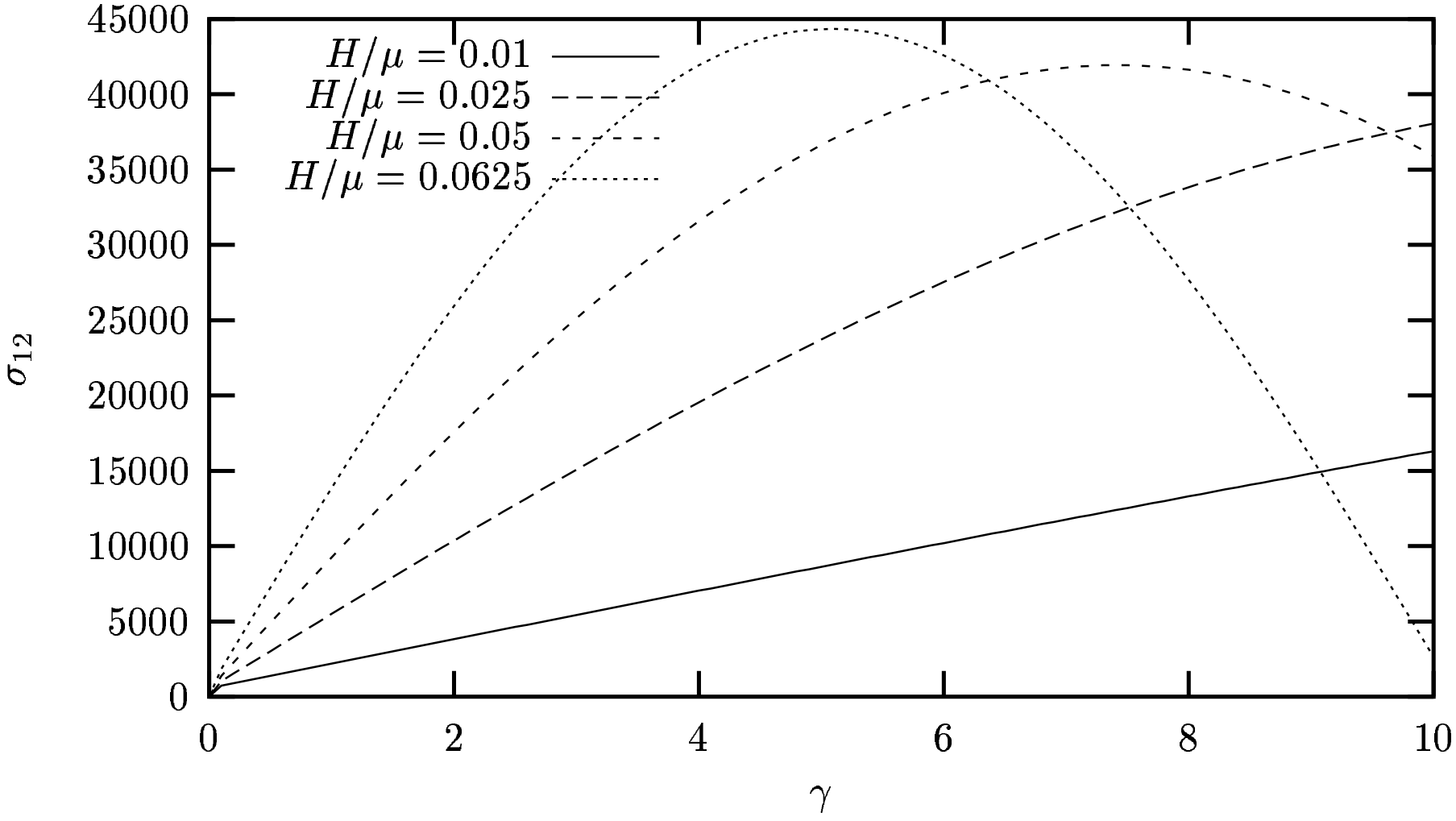
$$\begin{cases} \sigma_{11} = -\sigma_{22} = \dfrac{C}{D^2+3}(1 - \exp(-\dfrac{D}{\sqrt{3}}\gamma)(\cos\gamma + \dfrac{D}{\sqrt{3}}\sin\gamma)) \\ \sigma_{12} = \dfrac{C}{3}\exp(-\dfrac{D}{\sqrt{3}}\gamma)\sin\gamma + \dfrac{DC}{\sqrt{3}(D^2+3)}(1 - \exp(-\dfrac{D}{\sqrt{3}}\gamma)(\cos\gamma + \dfrac{D}{\sqrt{3}}\sin\gamma)) + \dfrac{R}{\sqrt{3}} \end{cases}$$

Simple glide and plasticity



kinematic hardening

Simple glide and plasticity



isotropic hardening

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Conclusions and recommendations

applications : forming, fracture... other?

- no religion on the choice of strain measures nor objective derivatives...
- experimental results are needed for identification...
- anisotropy evolution
- efficiency: objective local frames